Exercise 8

Find the area of a parallelogram bounded by the x-axis, the line $g(x) = 2$, the line $f(x) = 3x$, and the line parallel to $f(x)$ passing through $(6, 1)$.

Solution

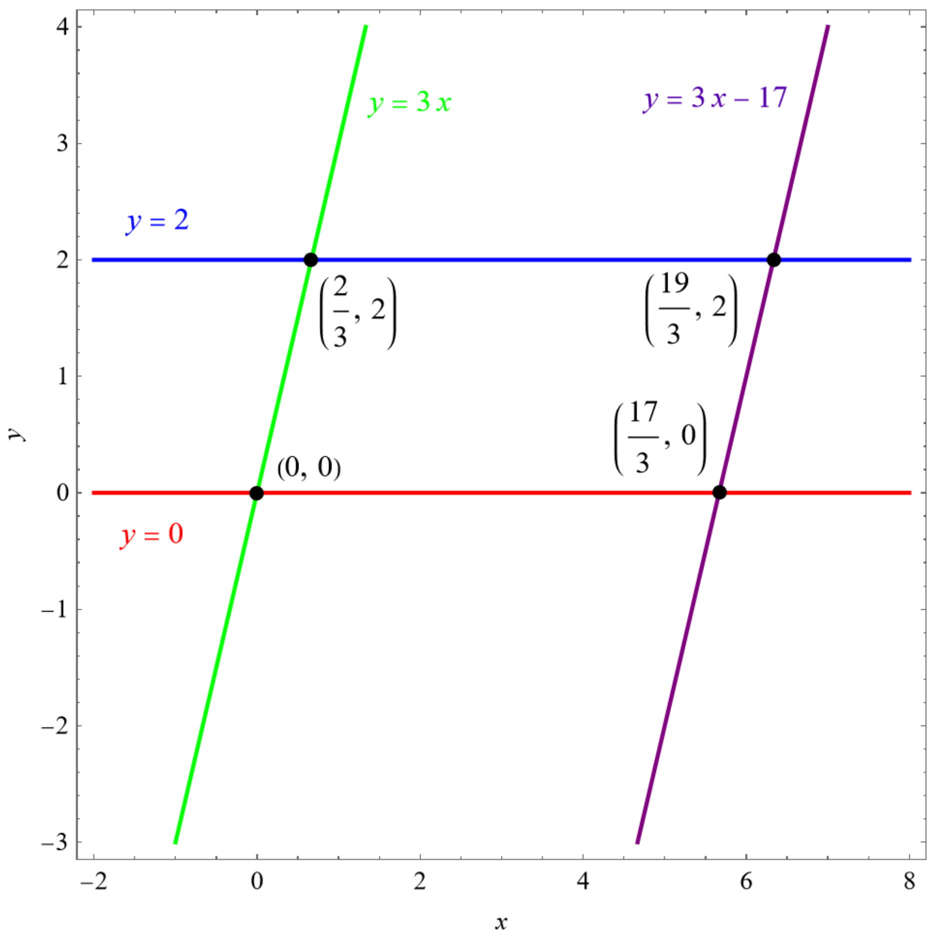
Start by writing equations of the lines that are given. The equation for the x-axis is $y = 0$, $y = 2$ is given, $y = 3x$ is given, and the line parallel to $f(x)$ has the same slope (3) with an equation given by the point-slope formula.

$$y - 1 = 3(x - 6)$$

$$y - 1 = 3x - 18$$

$$y = 3x - 17$$

Now graph all of them.



The area of the enclosed parallelogram is

$$A = \int_0^2 \left(\frac{y + 17}{3} - \frac{y}{3}\right) dy = \int_0^2 \left(\frac{17}{3}\right) dy = \frac{17}{3}(2 - 0) = \frac{34}{3}.$$

The point of intersection on the top left is found by solving the linear equations simultaneously.

$$y = 2 \quad \text{and} \quad y = 3x$$

$$2 = 3x$$

$$\frac{2}{3} = x$$

The top left point of intersection is $\left(\frac{2}{3}, 2\right)$. The top right point of intersection is found similarly.

$$y = 2 \quad \text{and} \quad y = 3x - 17$$

$$2 = 3x - 17$$

$$19 = 3x$$

$$\frac{19}{3} = x$$

The top right point of intersection is $\left(\frac{19}{3}, 2\right)$. The bottom right point of intersection is found similarly.

$$y = 0 \quad \text{and} \quad y = 3x - 17$$

$$0 = 3x - 17$$

$$17 = 3x$$

$$\frac{17}{3} = x$$

The bottom right point of intersection is $\left(\frac{17}{3}, 0\right)$.